**Course: Algorithm  
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Homework: Lab 4**

I referenced the **Lecture 3 – Probability.pdf, page 13 – 14**

*p* is probability of the success

*q* is probability of the failure

Expected number of trials to get a success when trying *m* times is:

S = 1.p + 2.q.p + 3.q2.p + … + m.qm-1.p

⬄ S.q = q.p + 2.q2.p + … + (m – 1).qm-1.p + m.qm.p

⬄ S – S.q = p + p.q + q2.p + … + qm-1.p – m.qm.p

⬄ S(1 – q) = p(1 + q + q2 + … + qm-1 – m.qm)

⬄ S.p = p( - m.qm)

⬄ S = - m.qm **(1)**

1. **Question 1**
2. *What is the average number of array locations to inspect to find a D*

We have 4 letter A, B, C, D. Let say, D is a success, [A, B, C] is a failure.

So, P(Success) = ¼, P(Failure) = ¾

Expected number of array locations to inspect to find a D – success is:

By using formula **(1)**, in this case we have m = 10

S= - 10.(10 = 4 – 4.(3/4)10 – 10.(3/4)10 = 4 – 14.(3/4)10 3.2116

1. *Calculate expected value of Z – random variable that indicate the index of D(success) in the array of 10 elements*

Z can be any of [0,1,2,3,4,5,6,7,8,9,10],

P(Z=0) = ¾ (the failure)

We have 10 positions for success = ¼, so each P(Z!=0) = (¼).(1/10) = 1/40

E(Z) = 0.(3/4) + 1.(1/40) + 2.(1/40) + 3.(1/40) + … + 10.(1/40)

= (1/40).(1 + 2 + 3 + … + 10) = (1/40) . = 11/8 1.375

1. **Question 2**
2. *Average number of array locations to inspect to find 10 Ds*

By using formula **(1)**, in this question m = 100. Expected number of array locations for 1 D is

S = - 100.(3/4)100 = 4 – 104.(3/4)100 4

In order to get k = 10 D = 10.4 = 40

1. *The average number of array locations to inspect to find k Ds*

( - *m*.qm).k

Size of the array is 100, so ( - *m*.qm).k = ( – *100.*q100).k (*because q100 is approximate 0)*

1. *Average time complexity to find k D in an array*

We have the average number of array locations to inspect to find k D is (size of array is 100, so q100is approximate zero)

Tavg =

1. **Question 3 -** Prove: 1 + ½ + 1/3 + … + 1/n = O(log*n*)

n = 7

1 + (1/2 + 1/3) + (1/4 + 1/5 + 1/6 + 1/7) < 1 + (1/2 + ½) + (1/4 + ¼ + ¼ + 1/4) = 3 = log(7 + 1) 🡺 Holds

So, 1 + (½ + ½) + (¼ + ¼ + ¼ + ¼) + … = log(n+1)

S = 1 + (1/2 + 1/3) + (1/4 + 1/5 + 1/6 + 1/7) + … + 1/n + 1/n+1

As we can see (1/2 + ½) > (1/2 + 1/3); (1/4 + ¼ + ¼ + ¼) > (1/4 + 1/5 + 1/6 + 1/7), so (1/k + … + 1/k) > (…+ 1/n + 1/n + 1)

So S **<** 1 + (1/2 + ½) + (1/4 + ¼ + ¼ + ¼) + … = log(n + 1)

1. **Question 4 –** Find the sum: ½ + 2/4 + 3/8 + 4/16 + 5/32 + …

S = ½ + 2/4 + 3/8 + 4/16 + 5/32 + …

S/2 = ¼ + 2/8 + 3/16 + 4/32 + …

S – S/2 = ½ + ¼ + 1/8 + 1/16 + 1/32 + …

⬄ S(1 – ½) = ½ []

⬄ S =

⬄ S = 2(1 -

When n goes to big, S = 2